RED SHIFT –MAGNITUDE RELATION

$\mathcal{K}$ correction

The magnitude of an object is determined by

$$m_b = -2.5 \log F_b + \mathrm{const}$$

where F is flux, which is related with absolute luminosity L_b of the object as

$$F_b = L_b / 4\pi l_b^2$$

So, we obtain

$$m_b = 5\log l_b(z) + M_b + 25$$

$$m_b - M_b = m - M - \mathcal{K} - A$$

A is factor of extragalactic absorption and
$\mathcal{K}$ is correction factor to a fixed
bandwidth

$$\mathcal{K} = 2.5\log(1+z)$$

One can determine also the flux at given frequency, which is related with absolute luminosity at this frequency L_ν of the object as

$$F_\nu = L_\nu / 4\pi l_\nu^2$$

where l_ν is monochromatic distance at this frequency

Suppose that the absolute luminosity is

$$L_\nu \propto \nu^\alpha$$

The flux observed at a frequency of ν in the given frequency band, is emitted at a higher frequency and within a wider frequency band

$$\Delta\nu_o = \Delta\nu_e / (1+z)$$

which together gives a correction in the form of a multiplier

$$(1+z)^{1+\alpha}$$

for the observed flux and the square root of this factor for monochromatic distance

$$l_\nu = l_b (1+z)^{-(1+\alpha)/2}$$

That is $\mathcal{K}$ correction for extragalactic radiosources.

In optical astronomy the introduction of $\mathcal{K}$ correction is much more difficult. That is because of presence the strong spectral lines, tilt of spectra, and wide optical bands.

At high redshift, optical imaging samples the rest frame ultraviolet light in galaxies. Might this affect the observed morphologies of high redshift galaxies? Could the unusual high redshift morphologies in the HDF be due to the different rest frame wavelengths at high redshift?

The answer is YES, and it is called sometimes morphological $\mathcal{K}$ correction.

The difference between observed and rest frame wavelengths changes the observed flux of high redshift objects, it is known as $\mathcal{K}$ correction.

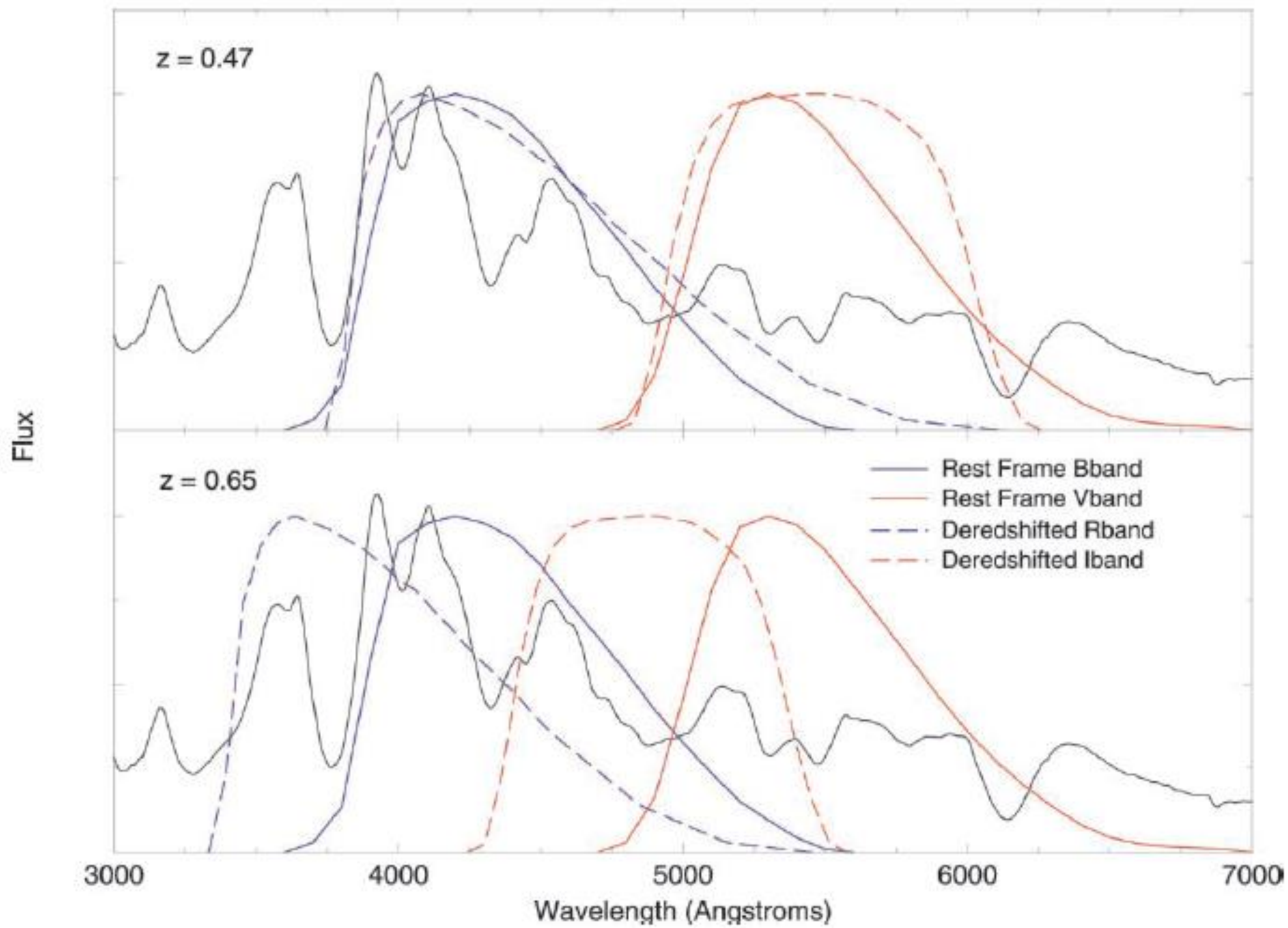
For instance, at submm and mm wavelength range galactic spectra are dominated by RJ slope of the thermal emission. The $\mathcal{K}$ correction is very strong and can make a distant galaxy **brighter** then a closer identical galaxy.

K-Corrections and Extinction Corrections for Type Ia Supernovae

PETER NUGENT, ALEX KIM, AND SAUL PERLMUTTER

The magnitude of an SN Ia in filter y can be expressed as the sum of its absolute magnitude M_x, cross-filter K-correction K_{xy}, distance modulus μ, and extinction due to dust in both the host galaxy A_x and our Galaxy A_y:

$$m_y(t(1+z)) = M_x(t, s) + K_{xy}(z, t, s, A_x, A_x) + \mu(z, \Omega_M, \Omega_\Lambda, H_0) + A_x(t) + A_y(t). \quad (1)$$


z = 0.47
z = 0.65
Rest Frame Bband
Rest Frame Vband
Deredshifted Rband
Deredshifted Iband
Flux
3000
4000
5000
6000
7000
Wavelength (Angstroms)

The cross-filter K-corrections presented in this paper were calculated as they were in KGP96:

$$K_{xy}^{\mathrm{counts}} = -2.5 \log \left(\frac{\int \lambda Z(\lambda) S_x(\lambda) d\lambda}{\int \lambda Z(\lambda) S_y(\lambda) d\lambda} \right) + 2.5 \log (1 + z)$$

$$+2.5 \log \left(\frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \lambda F[\lambda/(1 + z)] S_y(\lambda) d\lambda} \right) . \qquad (4)$$

Here $F(\lambda)$ is the spectral energy distribution (SED) at the source (in this case a supernova), $S_x(\lambda)$ and $S_y(\lambda)$ are the effective transmission of the x and y bands, $Z(\lambda)$ is an idealized local stellar spectral energy distribution for which $U = B = V = R = I = 0$ in the photometric system used here, and K_{xy} is defined as in equation (1). We have calculated the K-corrections

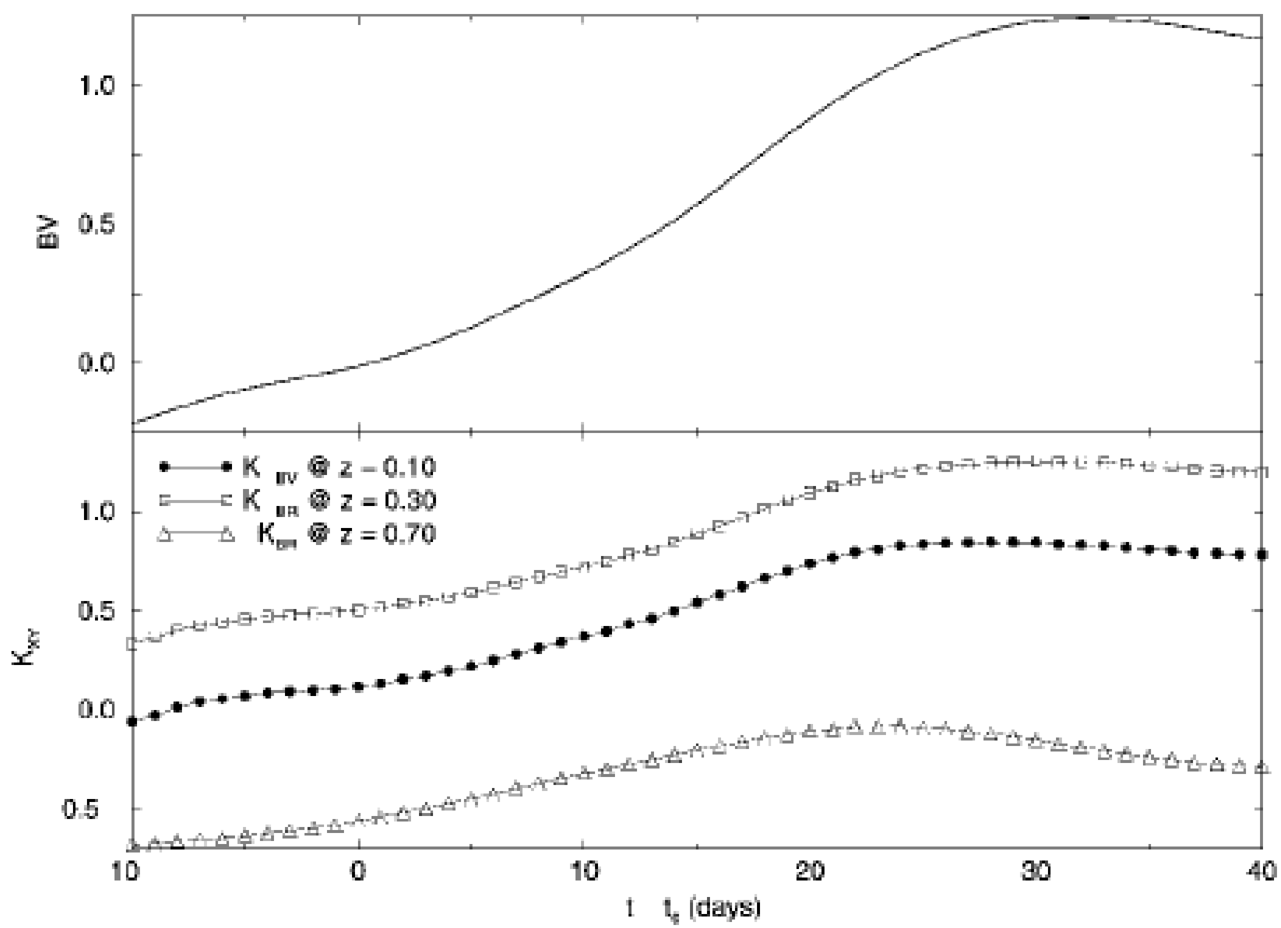


FIG. 3.—*Top panel*: Rest-frame $B-V$ color curve for an $s = 1.0$ SN Ia. *Bottom panel*: K-corrections for an $s = 1.0$ SN Ia at a variety of redshifts. Note the strong similarity between the color curve and the K-corrections.

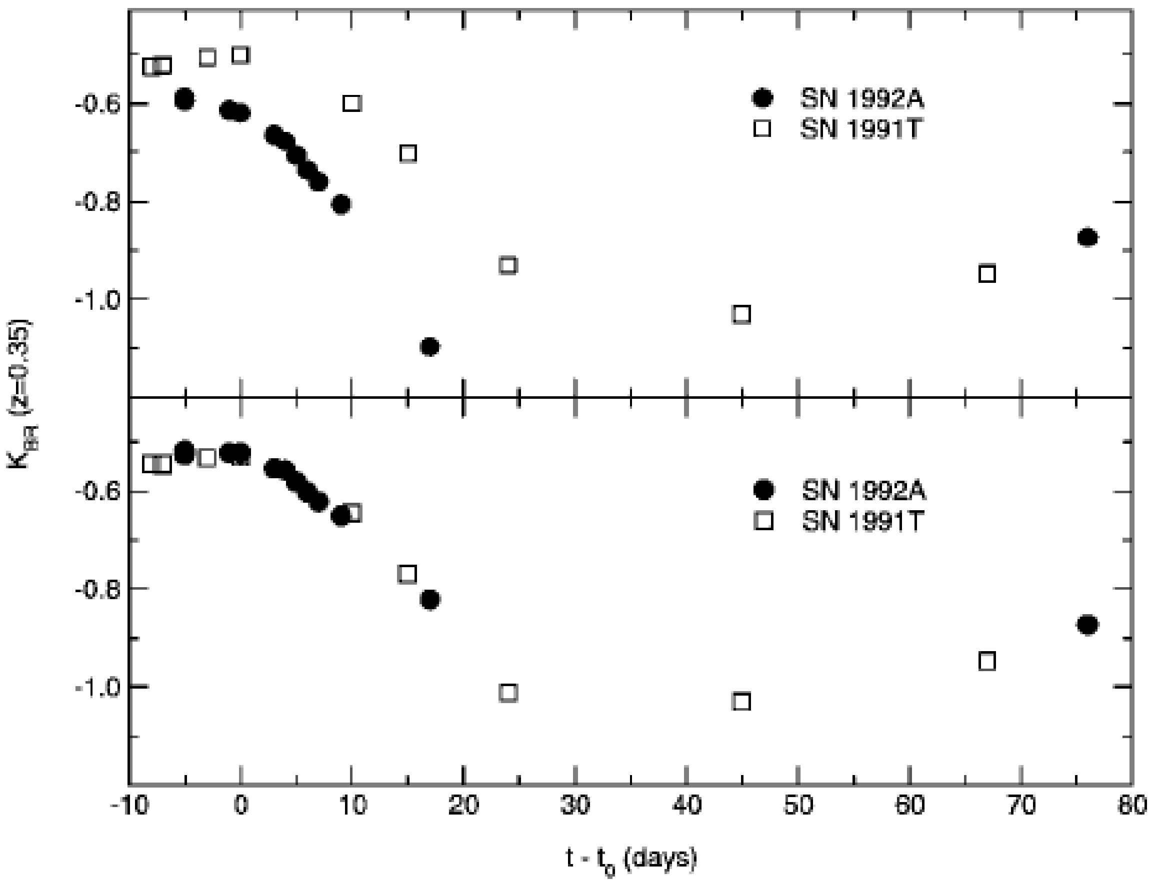


FIG. 4.—K_{BR} at $z = 0.35$ for SN 1992A (*black circles*) and SN 1991T (*red squares*). *Top panel*: *K*-corrections as would be observed for these spectroscopically distinct SNe Ia ($s = 0.80$ and 1.09, respectively). *Bottom panel*: Slope correction was applied to each of the supernova's spectra so that they would have the same $B-V$ colors as an $s = 1.0$ SN Ia at every epoch.

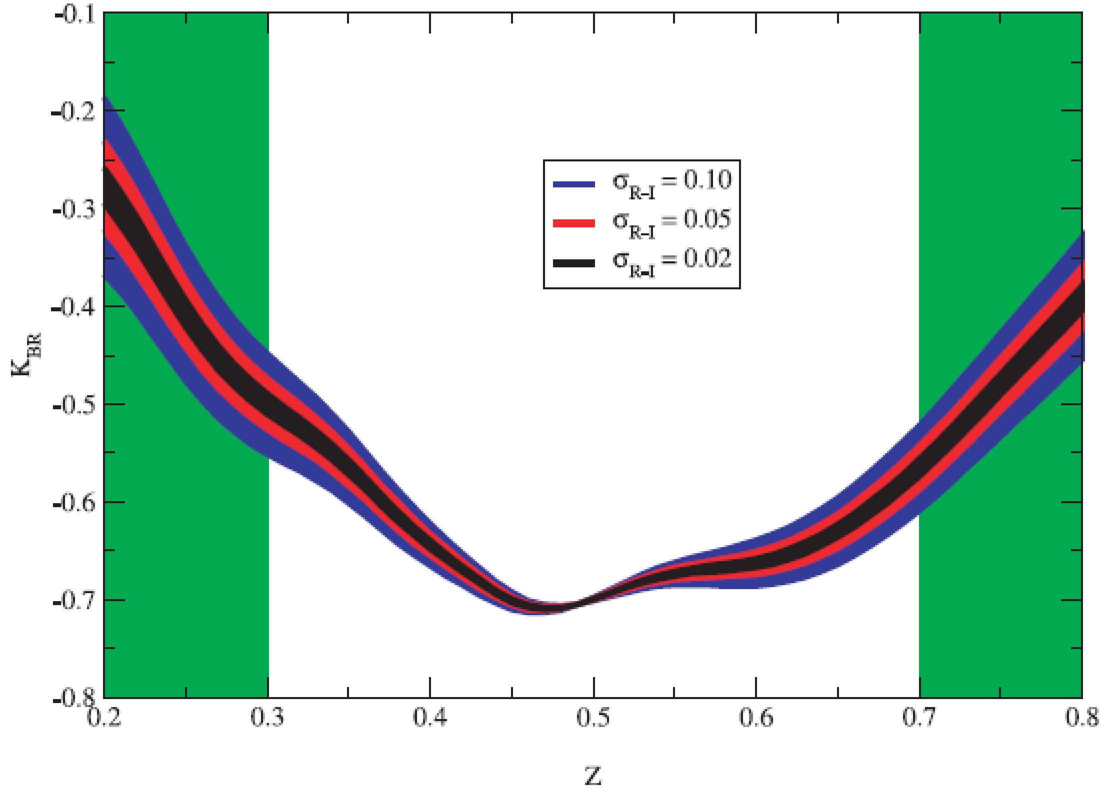


FIG. 7.—K_{BR} as a function of z for three different cases in which we assign an uncertainty in the observed $R-I$ color of 0.02 (*black*), 0.05 (*red*), and 0.10 (*blue*). Note how the uncertainty in K_{BR} vanishes at $z = 0.49$, where the rest-frame B- and deredshifted R-band filters nicely overlap, regardless of the uncertainty in the color measurement. In practice, one does not use K_{BR} in the green regions, where a switch is made to a more appropriate filter.

SOURCE COUNTS

The “SOURCE COUNTS” was one of the first test in cosmology. It also was related with so called “Olbers paradox”. This paradox was formulated by the Dutch astronomer H.Olbers. The first formulation was: if our Universe is infinite in space and time, then every line of sight should intersect with a star surface. So why the night sky is dark, not as bright as our Sun?

Let us consider toy model of source distribution in cosmos. They are randomly and homogeneously distributed over space and have equal luminosities. Apparent flux of the source which has distance r is

$$F = \frac{L}{4\pi r^2} \quad \text{Euclidean 3D space}$$

So, the sources which have apparent flux greater than F_0 are closer than r_0 :

$$r_0 = \left(\frac{L}{4\pi F_0} \right)^{1/2}$$

The number of sources which with $F > F_0$ is:

$$N(F > F_0) = \frac{4\pi}{3} n_0 \left(\frac{L}{4\pi F_0} \right)^{3/2}$$

where n_0 is average density of sources

One can differentiate $N(F_0)$ with respect to F_0 and obtain :

$$\frac{d \lg N}{d \lg F_0} = -\frac{3}{2}$$

One can introduce the density of sources $n(F)$ per interval $F, F + dF$:

$$n(F) = -\frac{dN}{dF}$$

and :

$$\frac{d\lg n(F)}{d\lg F} = -\frac{5}{2}$$

Let us consider the mathematical properties of this distribution. First of all it is very useful to calculate its mean, r.m.s. and other moments:

mean

$$\langle F \rangle = \int_0^{\infty} n(F) F dF = \int_{F_{\min}}^{F_{\max}} n(F) F dF$$

$$n(F) = n_0 \frac{F_{median}^{3/2}}{F^{5/2}}, \qquad \langle F \rangle = 2 n_0 F_{median}^{3/2} \left(\frac{1}{F_{\min}^{1/2}} - \frac{1}{F_{\max}^{1/2}} \right)$$

$$\text{when} \quad F_{\min} \to 0 \ \text{ mean value } \langle F \rangle \to \infty$$

It is Olbers paradox, The mean value flux is diverges, the brightness of the sky becomes infinite.

Let calculate r.m.s.

$$\left\langle F^2 \right\rangle = \int n(F) F^2 dF = 2 n_0 F_{median}^{3/2} \left(F_{\max}^{1/2} - F_{\min}^{1/2} \right)$$

$$\text{when} \quad F_{\max} \to \infty \ \text{ r.m.s. value } \langle F^2 \rangle \to \infty$$

From it follows, that r.m.s. value is determined by the most powerful and closest source in the sky.

The main conclusion of these results is: the sky should have the temperature of a source,

T=6 000 ^{0}K.

The resolution of this paradox is the evolution of the Universe

The number of sources is equal to product of source density n_0 by volume. The volume of a sphere is equal to

$$\mathscr{V} = \frac{4\pi}{3} r^3(z) \quad \text{where}$$

$r(z)$ is radius of comoving sphere,

i.e. cosmic distance $r(z) = \dfrac{l_b}{1+z}$

where $$l_b = \sqrt{\frac{L_b}{4\pi F_b}}$$

So, we can write

$$N(F_b) = \frac{4\pi}{3} n_0 \left(\frac{L_b}{4\pi F_b} \right)^{3/2} (1+z)^{-3}$$

where

$$z \approx \frac{H_0}{c}\sqrt{\frac{L_b}{4\pi F_b}}$$

Substituting this equation into *N(F)* we obtain

$$N(F_b) = \frac{4\pi}{3} n_0 \left(\frac{L_b}{4\pi F_b}\right)^{3/2} \left\{1 - \frac{3H_0}{c}\left(\frac{L_b}{4\pi F_b}\right)^{1/2} + \cdots\right\}$$

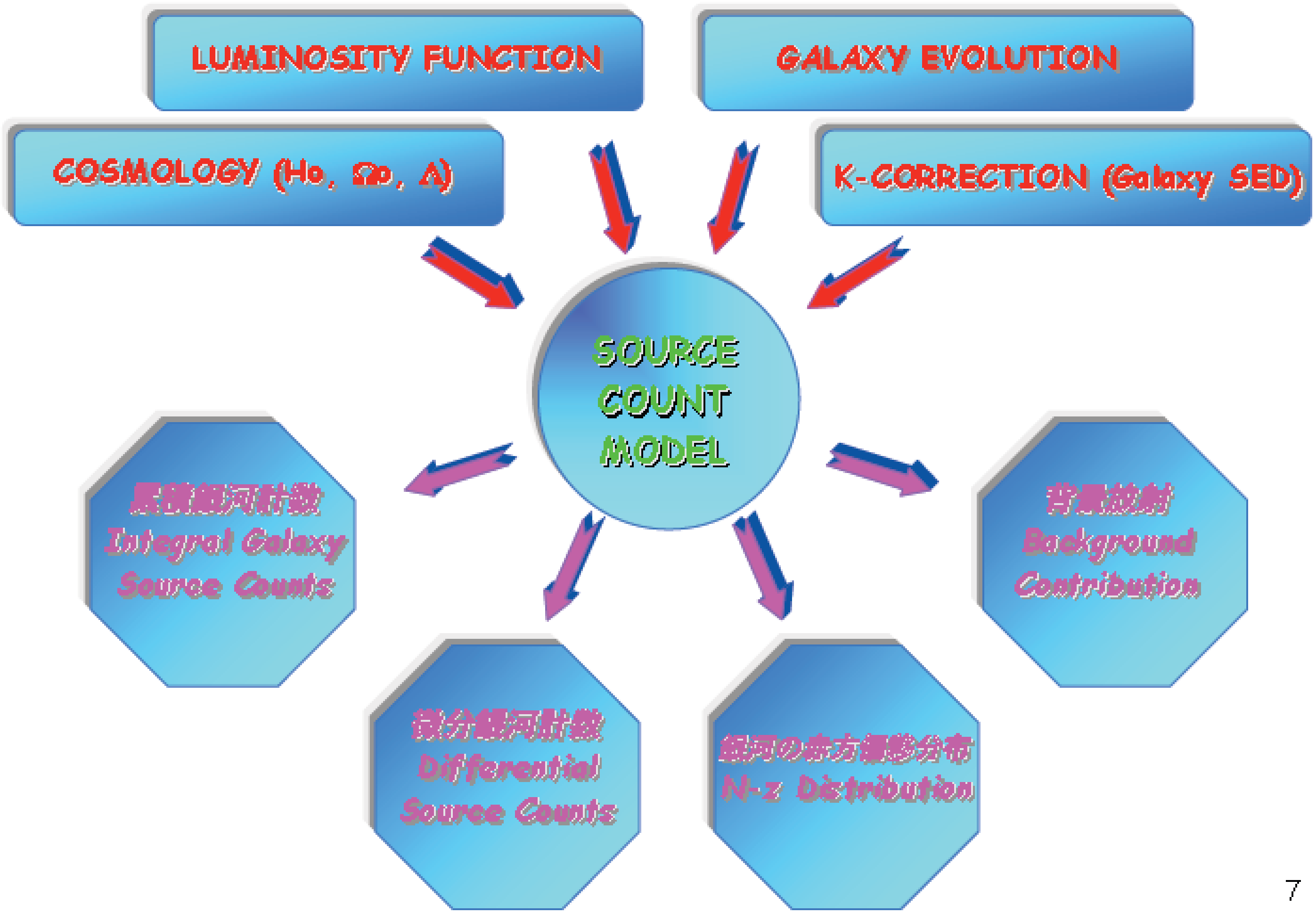
or

$$\frac{d\lg N}{d\lg F} = -\frac{3}{2} + \frac{3}{2}\frac{H_0}{c}\sqrt{\frac{L_b}{4\pi F_b}}$$

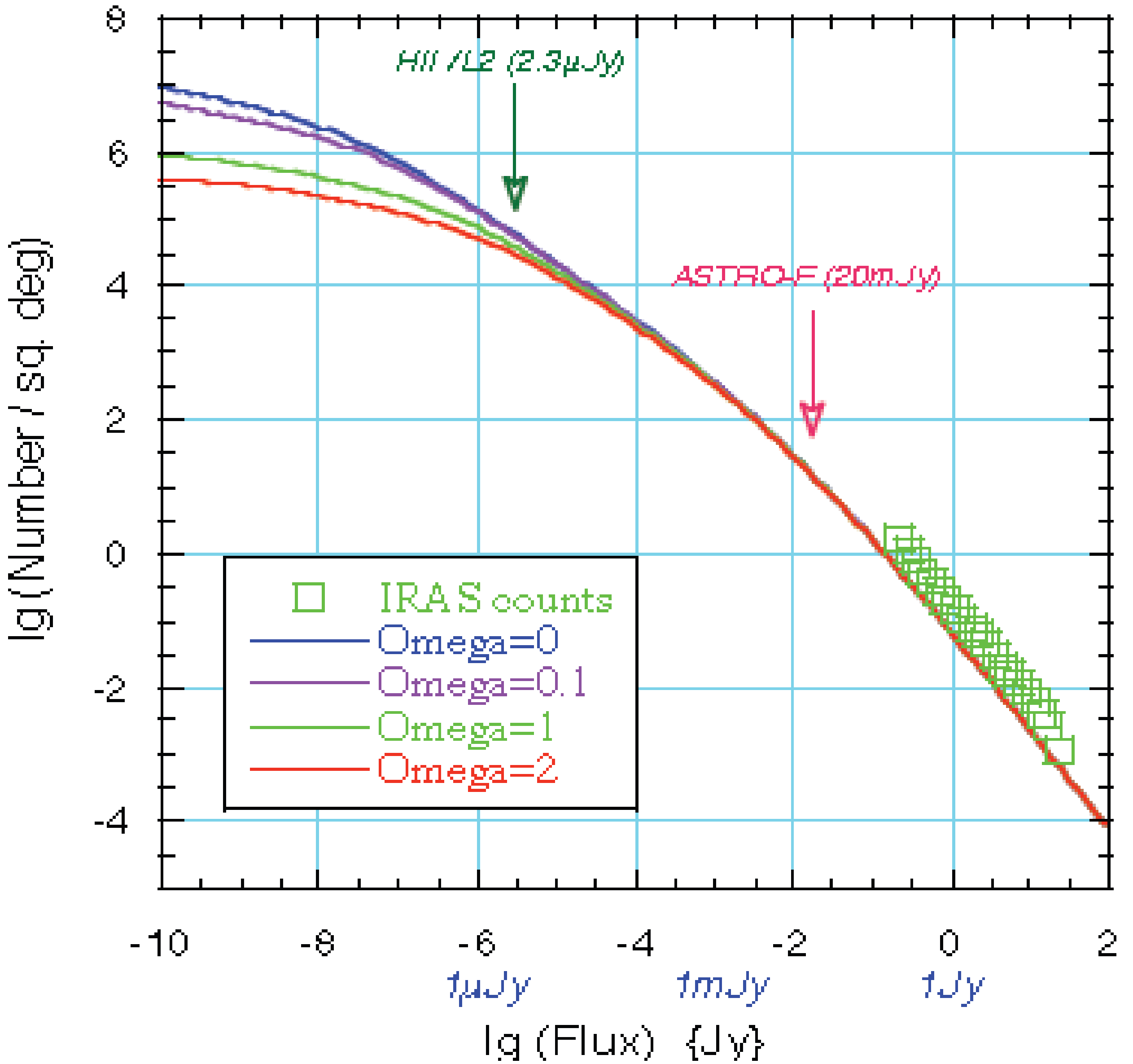
$$N_\upsilon(S) = \int_0^\infty dL \int_0^{z(L,S)} \phi(L,z)\, dV\, d\Omega_*$$

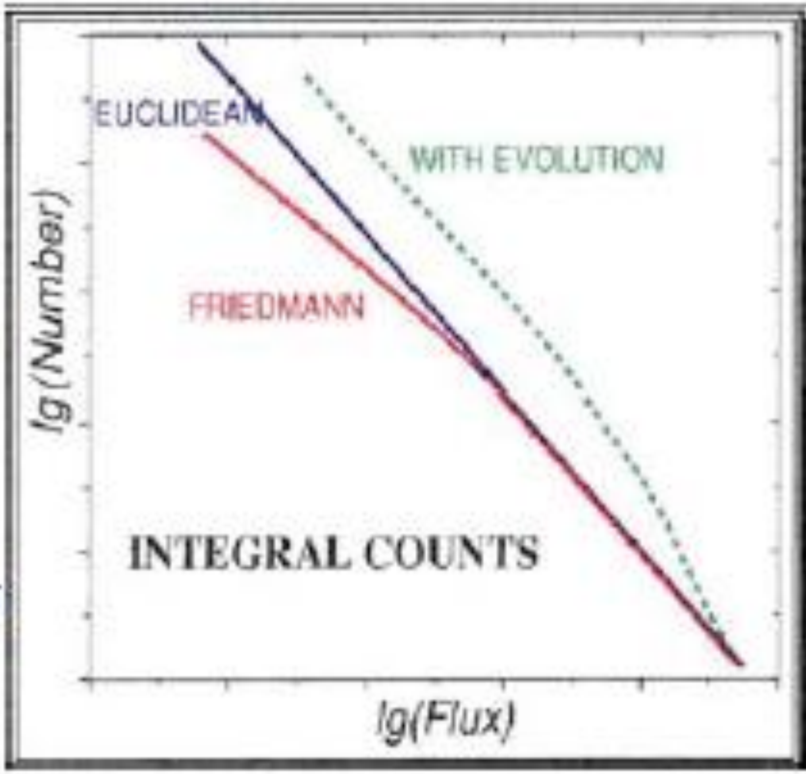
C.Pearson

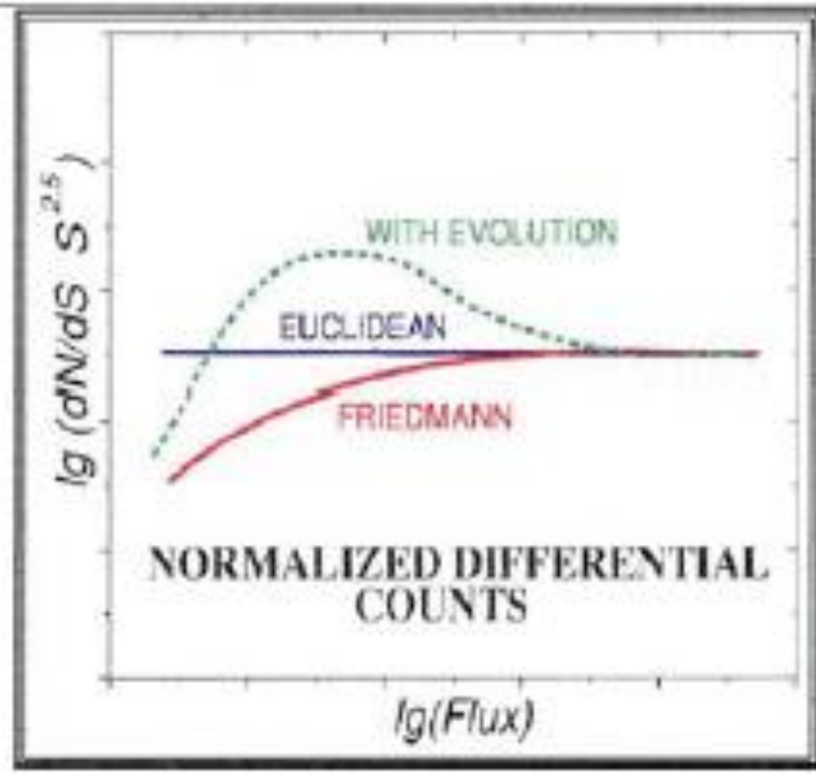
Derivation of Source Counts

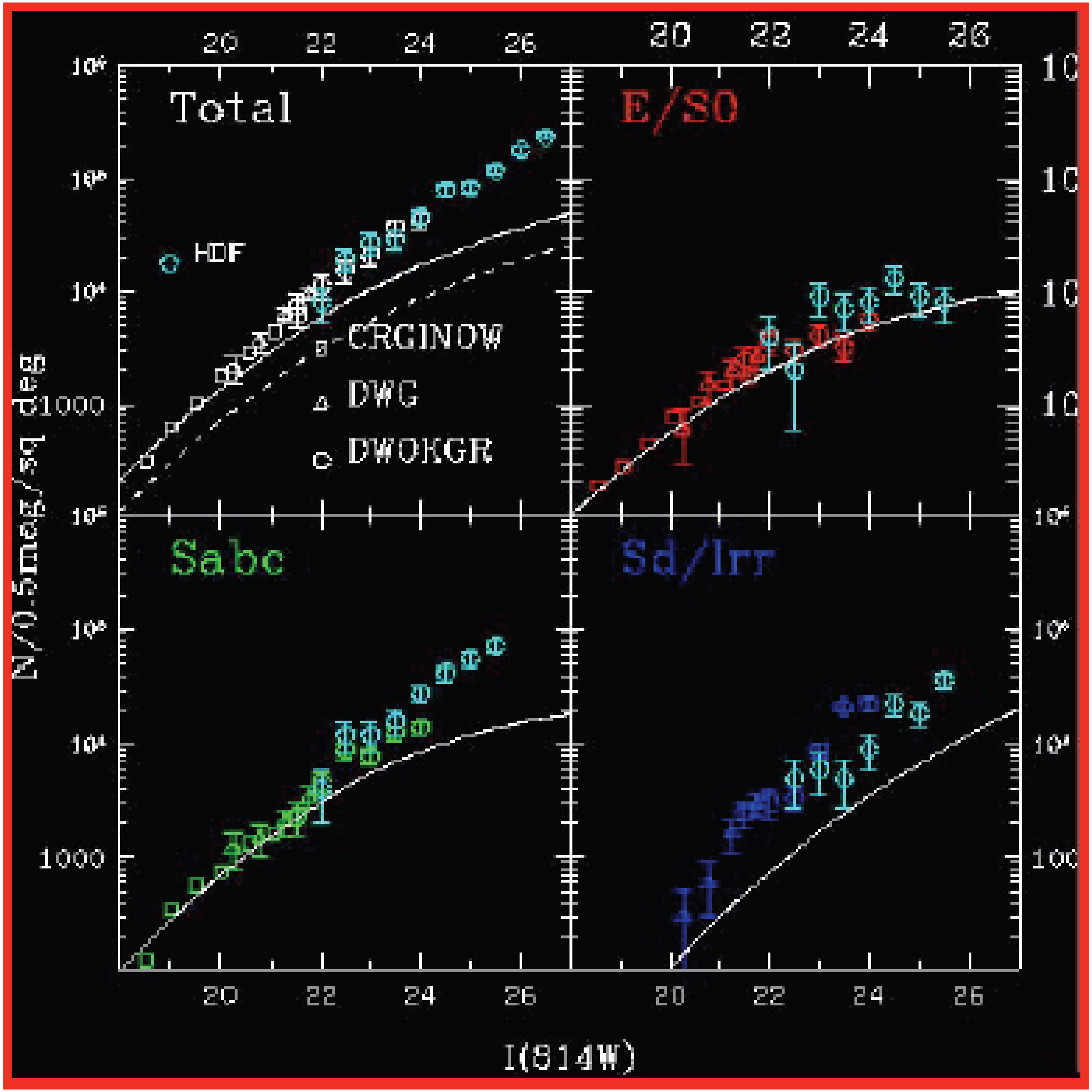


Integral Source Counts at 60μm




EUCLIDEAN
WITH EVOLUTION
FRIEDMANN
INTEGRAL COUNTS
lg(Number)
lg(Flux)


WITH EVOLUTION
EUCLIDEAN
FRIEDMANN
NORMALIZED DIFFERENTIAL COUNTS
lg (dN/dS S^2.5)
lg(Flux)



Faint blue galaxies were more numerous in the past , and may dominate the faint source counts